Glasgow Mathematical Journal 2017 vol.59 N2, pages 299-321

ON GORENSTEINNESS of HOPF MODULE ALGEBRAS

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Abstract

 Let H be a Hopf algebra with a bijective antipode, A an H-simple H-module algebra finitely generated as an algebra over the ground field and module-finite over its centre. The main result states that A has finite injective dimension and is, moreover, Artin-Schelter Gorenstein under the additional assumption that each H-orbit in the space of maximal ideals of A is dense with respect to the Zariski topology. Further conclusions are derived in the cases when the maximal spectrum of A is a single H-orbit or contains an open dense H-orbit.

http://dx.doi.org/10.1017/S0017089516000185

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